

Entropic force and its cosmological implications

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Abstract

We investigate a possibility of realizing the entropic force into the cosmology. A main issue is how the holographic screen is implemented in the Newtonian cosmology. Contrary to the relativistic realization of Friedmann equations, we do not clarify the connection between Newtonian cosmology and entropic force because there is no way of implementing the holographic screen in the Newtonian cosmology.

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1 Introduction

Since the discovery of the laws of black hole thermodynamics [1], Bekenstein [2] and Hawking [3] have suggested a deep connection between gravity and thermodynamics, realizing black hole entropy and Hawking radiation. Later on, Jacobson [4] has demonstrated that Einstein equations (describing relativistic gravitation) could be derived by combining general thermodynamic pictures with the equivalence principle. Padmanabhan [5] has observed that the equipartition law for horizon degrees of freedom combined with the Smarr formula leads to the Newton's law of gravity.

Recently, Verlinde has proposed the Newtonian force law as an entropic force (non-relativistic version) by using the holographic principle and the equipartition rule [6]. If it is proven correct, gravity is not a fundamental interaction, but an emergent phenomenon which arises from the statistical behavior of microscopic degrees of freedom encoded on a holographic screen. In other words, the force of gravity is not something ingrained in matter itself, but it is an extra physical effect, emerging from the interplay of mass, time, space, and information.

After his work, taking the apparent horizon as a holographic screen (HS) to derive the Friedmann equations [7], derivation of the Friedmann equations using the equipartition rule and unproved Unruh temperature [8, 9], the modified equipartition rule to discuss the large scale universe [10], and the correction to the entropic force by the corrected-entropy [11] were investigated for cosmological purpose. The connection to the loop quantum gravity [12], the accelerating surfaces [13], holographic actions for black hole entropy [14], and application to holographic dark energy [15] were considered from the view of the entropic force. Furthermore, cosmological implications were reported in [16, 17, 18, 19, 20, 21, 22], an extension to the Coulomb force [23], and the symmetry aspect of the entropy force [24] were investigated. The entropic force was discussed in the presence of black hole [25, 26, 27, 28, 29]. The Schwarzschild spacetime was introduced to define the holographic screen properly [30] and the entropic force did not always imply the Newtonian force law when imposing the non-gravitational collapse condition [31].

However, one of urgent issues to resolve is to answer to the question of how one can construct a spherical holographic screen of radius R which encloses a source mass M located at the origin to understand the entropic force. This is a critical and important issue because the holographic screen (an exotic description of spacetime) originates from relativistic approaches to black hole [32, 33] and cosmology [34]. Verlinde has introduced this screen by analogy with an absorbing process of a particle around the event horizon of black hole. Considering a smaller test mass m located at Δx away from the screen

and getting the change of entropy on the screen, its behavior should resemble that of a particle approaching a stretched horizon of a black hole, as described by Bekenstein [2].

Before proceeding, we would like to mention what is the difference between Newtonian gravity and general relativity [35]. First Newtonian gravity is an action-at-a-distance, that is, the gravitational influence propagates instantaneously ($c \rightarrow \infty$), implying the violation of causality. Second, Newtonian gravity is ignorant of the presence of horizon where the relativistic effects are supposed to dominate. For example, the horizons are considered as either the event horizon of black hole or the apparent horizons in the Friedmann-Robertson-Walker (FRW) universe. Comparing Newtonian gravity and general relativity in cosmology is different than in the case of isolated, asymptotically flat systems [36]. For isolated systems, both Newtonian gravity and general relativity are well-defined. In contrast, while relativistic cosmology is well-defined, there is no unique way to accommodate Newtonian theory of cosmology because the Newtonian equations are only defined up to boundary terms which have to be specified at all times.

Hence it is not easy to implement the entropic force into the cosmological setting. In the literatures [7, 9, 11], the authors did not mention explicitly how the entropic force (3) works for the cosmological purpose. It seems that the entropic force is not realized in the Newtonian cosmology unless the holographic screen is clearly defined.

In this work, we investigate intensively how the Newtonian cosmology is realized from the Poisson equation and Euler equation, the energy consideration, and spherical cavity together with the cosmological principle. If the boundary surface enclosing the cavity filled with the dust matter (or a source mass M) were replaced by the holographic screen to which the equipartition rule and the holographic principle are applied, the entropic force would derive the evolution of the dust matter-dominated universe.

2 Entropic force

When a test particle with mass m is close to a surface \mathcal{S} (holographic screen) with distance Δx (compared to the Compton wave length $\lambda_m = \frac{\hbar}{mc}$), the change of entropy on the holographic screen takes the form [6]

$$\Delta S = 2\pi k_B \frac{\Delta x}{\lambda_m} \rightarrow 2\pi m \Delta x \quad (1)$$

in the natural units of $\hbar = c = k_B = 1$ and $G = l_{pl}^2$. Considering that the entropy of a system depends on the distance Δx , an entropic force F_{ent} could be arisen from the analogy of the biophysics

$$F_{ent} \Delta x = T \Delta S \quad (2)$$

which may be considered as an indication that the first law of thermodynamics is realized on the holographic screen. Plugging (1) into (2) leads to an important connection between entropic force and temperature on the holographic screen

$$F_{ent} = 2\pi mT. \quad (3)$$

It implies that if one knows the temperature T on the holographic screen, the entropic force is determined by (3). Therefore, a key step is to determine the temperature on the holographic screen. Let us assume that the energy E is distributed on a spherical shape of holographic screen with radius R and the mass M is located at the origin of coordinate as the source mass. Then, we may introduce the equipartition rule to define the temperature T [37, 5], the equality of energy and mass, and the holographic principle to give the number of states N , respectively, as

$$E = \frac{1}{2}NT, \quad (4)$$

$$E = M, \quad (5)$$

$$N = \frac{A}{G} \quad (6)$$

with the area of a holographic screen $A = 4\pi R^2$. These are combined to determine the temperature on the holographic screen

$$T = \frac{GM}{2\pi R^2}. \quad (7)$$

Substituting (7) into (3), the entropic force is realized as the Newtonian force law

$$F_{ent} = \frac{GmM}{R^2}. \quad (8)$$

On the other hand, considering that Unruh has proposed the connection between acceleration and temperature

$$T_U = \frac{a}{2\pi} \rightarrow T, \quad (9)$$

Eq.(3) leads to the second law of Newton

$$F_{ent} = ma. \quad (10)$$

We remind the reader that T_U is the bulk temperature, while T is the boundary surface temperature.

3 Newtonian cosmology from the Poisson equation

We start with the Poisson equation for the Newtonian potential ϕ [38]

$$\phi_{,ii} = 4\pi G\rho \quad (11)$$

with $i = 1, 2, 3$. The continuity and Euler equations of fluid dynamics are given by

$$\dot{\rho} + \rho v_{i,i} = 0, \quad (12)$$

$$\dot{v}_i + \phi_{,i} + \frac{1}{\rho} p_{,i} = 0, \quad (13)$$

where v_i is the velocity field and p is the pressure. Homogeneity implies that the density and pressure are merely functions of time: $\rho(t)$ and $p(t)$. Also the velocity field is the same relative to all observers, which implies $v_i = V_{ij}(t)x_j$. Substituting the Poisson equation into the Euler equation leads to the fact that the Newtonian potential must take the form

$$\phi = a_{ij}(t)x_i x_j + a(t). \quad (14)$$

Hence, the Newtonian approximation of a homogeneous cosmology is determined by

$$a_{ii} = 4\pi G\rho, \quad \dot{\rho} + \rho V_{ii} = 0, \quad \dot{V}_{ij} + V_{ik}V_{kj} = a_{ij}. \quad (15)$$

Before we proceed, we mention that the relativistic (Friedmann-Robertson-Walker: FRW) cosmology is based on the isotropy and homogeneity. Thus, we will only consider the isotropic case where (15) becomes shear-free and rotation-free. However, in general, the Newtonian cosmology is anisotropic where there exist shear and rotation. To this end, we consider the following SO(3) decomposition

$$V_{ij} = \frac{1}{3}\theta\delta_{ij} + \sigma_{ij} + w_{ij}, \quad (16)$$

where

$$\theta = V_{ii}, \quad \sigma_{ij} = \frac{1}{2}(V_{ij} + V_{ji}) - \frac{1}{3}\theta\delta_{ij}, \quad w_{ij} = \frac{1}{2}(V_{ij} - V_{ji}). \quad (17)$$

Here, the trace part θ denotes the expansion, the trace-free symmetric tensor σ_{ij} represents the shear, and the anti-symmetric tensor w_{ij} describes the rotation. Plugging this decomposition into the Euler equation, and setting $\sigma_{ij} = 0$ and $w_{ij} = 0$ for a shear-free and rotation-free fluid, we obtain the equation for θ and the diagonalization of a_{ij} , respectively,

$$\dot{\theta} = -\frac{1}{3}\theta^2 - 4\pi G\rho, \quad a_{ij} = \frac{1}{3}a_{kk}\delta_{ij} \quad (18)$$

with continuity equation

$$\dot{\rho} + \rho\theta = 0. \quad (19)$$

Rewriting the expansion parameter $\theta = 3\frac{\dot{\tilde{R}}}{\tilde{R}}$ in terms of a Newtonian scale factor \tilde{R} , the solution to the continuity equation (19) is

$$\rho = \frac{C'}{\tilde{R}^3}, \quad (20)$$

with C' a constant.

Using (20) in the Euler equation, we see that the isotropic and homogeneous Newtonian cosmology is described by

$$a_{ii} = 4\pi G\rho, \quad (21)$$

$$\frac{\ddot{\tilde{R}}}{\tilde{R}} = -\frac{4}{3}\pi G\rho, \quad (22)$$

$$\rho = \frac{C'}{\tilde{R}^3}. \quad (23)$$

At this stage, we mention the FRW case whose equations are given by the Friedmann equation without the tilde (\sim) notation

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{R^2}, \quad (24)$$

and the Raychaudhuri equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + 3p\right), \quad (25)$$

from which one may obtain the Bianchi identity

$$\dot{\rho} + 3\left(\rho + p\right)\frac{\dot{R}}{R} = 0. \quad (26)$$

We note the difference that $R(t)$ is the relativistic scale factor in (24)-(26), while $\tilde{R}(t)$ is the Newtonian scale factor in the Newtonian cosmology (21)-(23).

We are now in a position to compare these two theories. The general relativistic theory is well-posed. Equations (24) and (25) are consistent with the Bianchi identity. In the Newtonian theory there is only one equation (21), and there is no completeness because (21) does not give (22) and (23), and nor is the theory well-posed. Also, notice that in the general relativistic theory, pressure occurs in the dynamics of the theory, whereas in the

Newtonian theory, pressure does not occur anywhere in the dynamics and is only defined through an equation of state. Equation (22) has the same form as the Raychaudhuri equation (at least when $p = 0$).

Let us answer to the question of how do $R(t)$ and $\tilde{R}(t)$ differ. Considering

$$a_{ii} = -3\frac{\ddot{\tilde{R}}}{\tilde{R}}, \quad a(t) = \dot{A}, \quad (27)$$

the Poisson equation (21) yields

$$\frac{\ddot{\tilde{R}}}{\tilde{R}} = -\frac{4}{3}\pi G\rho. \quad (28)$$

This is again the Raychaudhuri equation of general relativity for the dust matter $p = 0$. Hence, the general relativistic scale factor $R(t)$ is equivalent to the Newtonian scale factor $\tilde{R}(t)$. However, it is argued that Newtonian cosmology is applicable only when confined to a neighborhood of the observer, corresponding to distances which are small compared to the Hubble distance $d_H = 1/H$. Therefore, *it is problematic to define the apparent horizon as the holographic screen in the Newtonian cosmology*. For a flat spacetime, the apparent horizon occurs at $r_A = 1/H$.

Furthermore, the Newtonian theory suffers in that varying the equation of state $p = \omega\rho$ will have no effect on the outcome of the solutions for $\rho(t)$ and $\tilde{R}(t)$. This is because the pressure does not appear in the dynamical equations. Thus, we may at most reproduce the results of the matter-dominated case of general relativity. Finally, it is worth to note that the Friedmann equation (24) was missed in the Newtonian cosmology. This equation could be realized from another approach of Newtonian mechanics together with its energy consideration.

Consequently, the Euler equation (13) leads to the Raychaudhuri equation (22) for the description of Newtonian cosmology when combined with the cosmological principle.

4 Newtonian cosmology from energy

In this section, we will show how the Friedmann equation (24) comes out from the Newtonian mechanics. We propose that a system of the universe consists of a number N of galaxies with their mass m_p and position $\mathbf{r}_p(t) = r_p(t)\hat{\mathbf{r}}$ as measured from a fixed origin O [39]. Then the kinetic energy T of the system is given by

$$T = \frac{1}{2} \sum_{p=1}^N m_p \dot{r}_p^2. \quad (29)$$

The gravitational potential energy V is

$$V_g = -G \sum_{p < q}^N \frac{m_p m_q}{|\mathbf{r}_p - \mathbf{r}_q|}. \quad (30)$$

Then, the total energy E of this system is given by

$$E = \frac{1}{2} \sum_{p=1}^N m_p \dot{r}_p^2 - G \sum_{p < q}^N \frac{m_p m_q}{|\mathbf{r}_p - \mathbf{r}_q|}. \quad (31)$$

Suppose that the distribution and motion of the system is known at some fixed epoch $t = t_0$ as an initial condition. Invoking the cosmological principle of homogeneity and isotropy, the radial motion at any time t is then given by $r_p(t) = S(t)r_p(t_0)$ where $S(t)$ is a universal function of time which is the same for all galaxies and is related to the Newtonian scale factor. Substituting this into Eq.(31) leads to an energy relation

$$E = A\dot{S}(t)^2 - G\frac{B}{S(t)}, \quad (32)$$

where coefficients A and B are positive constants given by

$$A = \frac{1}{2} \sum_{p=1}^N m_p [r_p(t_0)]^2, \quad B = \sum_{p < q}^N \frac{m_p m_q}{|\mathbf{r}_p(t_0) - \mathbf{r}_q(t_0)|}. \quad (33)$$

Here B contains the gravitational configuration of the system at the initial time $t = t_0$. Eq.(32) is one form of the cosmological differential equation for a scale factor $S(t)$. If the universe is expanding, A -term decreases since the total energy remains constant as B -term decreases. Therefore, the expansion must slow down. Introducing the Newtonian scale factor with $S(t) = \mu\tilde{R}(t)$, Eq.(32) takes the form

$$\frac{\dot{\tilde{R}}^2}{\tilde{R}^2} = G\frac{C_1}{\tilde{R}^3} - \frac{k}{\tilde{R}^2}, \quad (34)$$

where the constants C_1 and k are defined by $C_1 = \frac{B}{\mu^3 A}$ and $k = -\frac{E}{\mu^2 A}$. When $E = 0$, μ is arbitrary. However, if $E \neq 0$, one can choose $\mu^2 = |E|/A$ so that $k = 1, 0, -1$. That is, the sign of E determines the evolution of the universe: for $E < 0$, it will contract, while for $E > 0$ it will expand. This equation is the same form of the Friedmann equation (24) of a relativistic cosmology. We mention that there exist ambiguities in determining the cosmological parameters C_1 and k . However, we note that the term in the left-hand side of Eq.(34) originates from the kinetic energy, the first term (last term) in the right-hand side come from the potential energy (total energy). In order to derive the Raychaudhuri equation, one may use the conservation of energy ($\dot{E} = 0$) to find

$$\frac{\ddot{\tilde{R}}}{\tilde{R}} = -\frac{GC_1}{2} \frac{1}{\tilde{R}^3}. \quad (35)$$

If $C_1 = \frac{8\pi}{3}C'$, the above equation leads to the Raychaudhuri equation (25) with $p = 0$. One may attempt to interpret (35) as the Newtonian force equation for a test mass with $m = 1$ on a S^2 of a proper radius of $\hat{r} = r\tilde{R}(t)$. Here, \tilde{R} is the Newtonian scale factor to describe the evolution of the matter-dominated universe. However, it is not clear that (35) is interpreted as the Newtonian force law.

Finally, we have derived the Friedmann equation (34) from the energy condition together with cosmological principle.

5 Newtonian force law on an expanding cavity

Let us introduce an expanding cavity of radius \hat{r} centered at O whose volume is $V = 4\pi\hat{r}^3/3$. According to the Gauss theorem in Newtonian mechanics (Birkoff's theorem in general relativity), the net gravitational effects of a uniform external medium on a spherical cavity is zero. In other words, the force acting on a test mass located at the boundary of $\partial V = S^2$ is the gravitational attraction from the matter internal to \hat{r} only, which may be considered as a point mass M at O. This is close to the situation to define the entropic force. *However, we never choose this boundary as the holographic screen.* Here, we follow the classical approach to finding the Newtonian force law. The mechanical energy of a test particle at the boundary is given by the sum of kinetic and gravitational potential energy as

$$U = \frac{1}{2}m\dot{\tilde{r}}^2 - \frac{GmM}{\tilde{r}^2} = \frac{1}{2}m\dot{\tilde{r}}^2 - \frac{4\pi}{3}Gm\rho\tilde{r}^2, \quad (36)$$

where the mass inside the cavity is

$$M = \rho V. \quad (37)$$

Rewriting the physical distance \tilde{r} in terms of the comoving distance r and the Newtonian scale factor \tilde{R} as

$$\tilde{r} = r\tilde{R}(t), \quad (38)$$

the energy conservation law leads to

$$U = \frac{1}{2}m\dot{\tilde{R}}^2 r^2 - \frac{4\pi}{3}Gm\rho\tilde{R}^2 r^2. \quad (39)$$

This can be rearranged into the Friedmann equation as

$$\frac{\dot{\tilde{R}}^2}{\tilde{R}^2} = \frac{8\pi G}{3}\rho - \frac{\tilde{k}}{\tilde{R}^2}, \quad (40)$$

where

$$\tilde{k} = -\frac{2U}{mr^2} \quad (41)$$

which is the same form as k in (34) when the correspondence is assumed to be

$$E \longleftrightarrow U, \quad \mu \longleftrightarrow r, \quad A \longleftrightarrow \frac{m}{2}. \quad (42)$$

Hence, we note that \tilde{k} is constant, implying that $U \propto r^2$. This means that while U is constant for a given particle, it changes if one looks at different comoving separations r .

Differentiating the energy conservation (36) with respect to time t ($\dot{U} = 0$), we have the Newtonian force law on m located at the boundary surface

$$m\ddot{\tilde{R}} = -\frac{GmM}{\tilde{R}^2}, \quad (43)$$

which states clearly that the only force on m is due to the mass M inside the cavity. In deriving (43), we used the constant of mass M in the cavity

$$\dot{M} = 0. \quad (44)$$

We note that dividing the Newtonian force law (43) by m leads to the Raychaudhuri equation (35) which is an accelerating equation for the Newtonian cosmology. This means that in the Newtonian cosmology, the Newtonian force law determines the acceleration of the cosmological evolution. One may consider that (43) is the Newtonian force law for a mass m which is circulating around the source mass M because the left hand side seems to be a centripetal force. However, for cosmological purpose, we consider the motion in the radial direction but not the motion in the tangential direction.

At this stage, we introduce the first law which takes the form

$$dE + pdV = TdS. \quad (45)$$

We apply the first law of thermodynamics to an expanding cavity of unit comoving radius with $r = 1$ filled by M . Using $E = M$, the first law leads to

$$dM = TdS \quad (46)$$

which for an isentropic expansion of $dS = 0$, it leads to (44) and finally

$$dM = 0 \rightarrow \dot{M} = 0 \rightarrow \dot{\rho} + 3\frac{\dot{\tilde{R}}}{\tilde{R}}\rho = 0, \quad (47)$$

where the last expression is simply the continuity equation (19) for the dust matter.

Consequently, we have found the Newtonian force law on the test particle m located at the boundary of an expanding cavity from the energy consideration together with the constant mass (continuity equation). We have considered regions smaller than the Hubble horizon ($\tilde{r} \ll 1/H$) and the expansion velocity are small ($v \ll c$) and thus, nonrelativistic dynamics are used. In this case, the first law of thermodynamics simply implies the continuity equation for the dust matter.

6 Entropic force and cosmological implications

In order to see what happens when the entropic force was introduced to describe the cosmology, let us consider a few of relevant works. For this purpose, a radical change should be made such a way that the boundary surface enclosing the spherical cavity is replaced by the holographic screen:

$$\text{boundary surface } (\partial\mathcal{V}) \longrightarrow \text{holographic screen (HS)}. \quad (48)$$

According to Ref.[9], the Raychaudhuri equation (22) was obtained by considering both the equipartition rule (4) and the (Unruh) temperature

$$T_* = \frac{a_r}{2\pi} = -\frac{r\ddot{R}}{2\pi} \quad (49)$$

on the boundary screen $\partial\mathcal{V}$ enclosing the spatial region of volume $\mathcal{V} = \frac{4}{3}\pi\hat{r}^3$ with $\hat{r} = rR(t)$. Here, $a_r = -d^2\hat{r}/dt^2$ is considered as the physical acceleration for a radial comoving observer located at a point of the boundary screen. However, it is well-known that the proper acceleration vanishes for a comoving observer in the relativistic FRW approach. Hence it is strange to introduce the (Unruh) temperature T_* in the nonrelativistic approach. Explicitly, in order to derive the Raychaudhuri equation (22), they have used the equipartition relation directly

$$E(=M) = \frac{NT_*}{2}. \quad (50)$$

However, the (Unruh) temperature T_* was not proven to be valid for this case. Padbanabhan [8] has shown that there is no simple justifications for defining T_* using the acceleration of geodesic derivation vector and thus, his successes of obtaining the Raychaudhuri equation must be considered as fortuitous. This suggests that the recovery of the Raychaudhuri equation from the equipartition rule together with the temperature T_* seems to be accidental. Otherwise, one has to explain why the equipartition rule with T_* is equivalent to the Raychaudhuri equation for the dust matter. This could not be

explained in view of the (nonrelativistic) entropic force because the equipartition rule was used to define the temperature on the holographic screen. We note that the temperature T_* is positive only for a deceleration of matter-dominated universe. On the other hand, T_* is negative for an acceleration of $\omega < -1/3$, which shows that T_* is not acceptable when extending other matter contents. Although they were succeeded in deriving the Friedmann equation (24) from the Raychaudhuri equation (25) by replacing the source mass M by the Tolman-Komar mass and integrating the resulting equation, it has nothing to do with a nonrelativistic force law of (2) and (3) because they have used the equipartition rule (50). Furthermore, it is not clear why the equipartition rule does provide the Raychaudhuri equation which corresponds to the Newtonian force law in the cosmological setting.

As a slightly different approach, the author [11] has imposed the Newtonian force law for a test particle m near the boundary screen $\partial\mathcal{V}$ directly

$$F = mr\ddot{R} = F_{ent}, \quad F_{ent} = -\frac{GMm}{R^2} \left[1 - \frac{\beta}{\pi} \frac{G}{R^2} - \frac{\gamma}{4\pi^2} \frac{G^2}{R^4} \right] \quad (51)$$

to find the corrected-Raychaudhuri equation by considering the corrected-entropy

$$S_c(A) = \frac{A}{4G} - \beta \ln \left[\frac{A}{4G} \right] + \gamma \frac{G}{A} + \dots \quad (52)$$

It seems that this approach mimics the Newtonian cosmology because (51) is the same as in (43) when disregarding the correction terms. However, the origin of the force (F_{ent}) is different from the Newtonian cosmology because he has used the equipartition rule and holographic principle prior to the derivation of the corrected-entropic force F_{ent} on the holographic screen.

The other is a derivation of Friedmann equation by using the apparent horizon as the holographic screen and taking into account the differential of equipartition rule (equivalently, the first law of thermodynamics) [7]. They have started with considering the relativistic FRW metric to define the apparent horizon. Hence, this is surely a relativistic approach. As was previously emphasized, it is unlikely to define the apparent horizon in the nonrelativistic Newtonian cosmology. This amounts to introducing the Schwarzschild spacetime to define a proper holographic screen [30]. If one introduces the apparent horizon r_A properly, thermodynamics is defined by giving the Hawking temperature

$$T_A = \frac{1}{2\pi r_A}, \quad r_A = \frac{1}{\sqrt{H^2 + k/R^2}} \quad (53)$$

with the Hubble parameter $H = \dot{R}/R$. In this case, the differential of the equipartition

rule is equivalent to the first law of thermodynamics as

$$dE_A = \frac{N_A}{2}dT_A + \frac{T_A}{2}dN_A = \frac{dr_A}{G} \longleftrightarrow dE_A = T_A dS_A \quad (54)$$

where

$$N_A = \frac{4\pi r_A^2}{G} = 4S_A. \quad (55)$$

Hence, the equipartition rule is nothing new but redundant. Supplying the perfect fluid as the energy-momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$ like

$$dE_A = 4\pi \tilde{r}_A^2 T_{\mu\nu} k^\mu k^\nu dt \quad (56)$$

they have led to the Friedmann equation (24) and then, the Raychaudhuri equation (25). However, this approach is not a non-relativistic approach and thus has nothing to do with the entropic force defined in (2) and (3), since the Friedmann equation was derived from the first law of thermodynamics (differential of equipartition rule but not equipartition rule itself).

Finally, we do not mention the entropic force applications to the accelerating universe [15, 21] and the inflation [40] because these issues are beyond the Newtonian cosmology.

7 Discussions

First of all, we wish to point out the difference between Newtonian gravity and general relativity [35]. Newtonian gravity is an action-at-a-distance which means that the gravitational influence propagates instantaneously, implying the violation of causality. Also, Newtonian gravity is ignorant of the presence of horizon where the relativistic effects are supposed to dominate. For example, the horizons are considered as either the event horizon of black hole or the apparent horizons in the FRW universe. It is suggested that the absence of horizon does not enable to implement the holographic screen in the Newtonian gravity (cosmology). The holographic principle (holographic screen) appears in the black hole which is formed after the gravitational collapse through the supernova explosion and in the FRW universe based on the cosmological principle when using the relativistic approach [41]. In this sense, the holographic principle seems to have nothing to do with the Newtonian gravity (cosmology).

It seems that the entropic force (2) based on the biophysics is not realized in the Newtonian cosmology unless the holographic screen is implemented. Actually, there is no justification (48) of taking the boundary surface (∂V) enclosing an expanding cavity V as

the holographic screen (HS) which contains space, time, and information. Presumably, if the holographic screen were introduced in the Newtonian mechanics, one would propose the equipartition rule on the holographic screen. This means that gravitational attraction could be the result of the way that information about material objects is organized in space. Accordingly, we could define the entropic force to reproduce the Newtonian force law. This is a way of realizing an entropic force as an emergent phenomena which arises from the statistical behavior of microscopic degrees of freedom encoded on the holographic screen [6]. However, this approach is too abstract to realize the entropic force. Thus, the Verlinde's proposition on the entropic force may be a hand-waving dimensional argument on the gravitational (cosmological) side.

In conclusion, we have not confirmed the connection between Newtonian cosmology and entropic force. We hope that the Newtonian cosmology may provide a simple testbed to prove the entropic force.

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